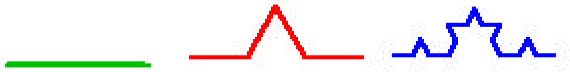
Cynthia Lanius

Fractal Properties

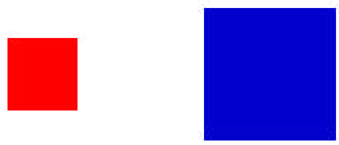


Self-Similarity

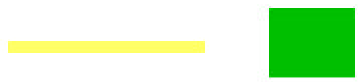
Table of Contents

Cats, canaries, or kangaroos are *similar* if they are alike in some way. In geometry though, similar means something very specific. Geometric figures are ***similar*** if they have the same shape. I don't mean two rectangles or two triangles, but *really* the same shape. For example:

The two squares are ***similar.***



The two rectangles are ***not similar.***



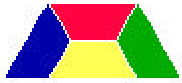
But the two rectangles below ***are similar.***

Look carefully at the last blue rectangle and you will see that it is 2 times as wide as the red rectangle and 2 times as long. We say that the sides are in proportion and the ratio is 2:1. Since the corresponding sides are in proportion (and the corresponding angles are also of equal measure), the figures are the same shape and are ***similar***.

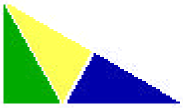
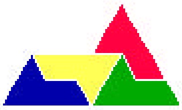
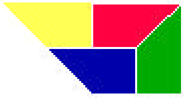
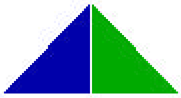
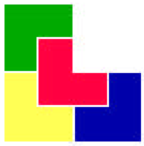
Consider similarity in another way. In order for one figure to be similar to another, you must be able to zoom in on one of the figures, and it will look exactly like the other one. (Of course you might have to rotate it or flip it).

Now how are figures ***self-similar?***

Many figures that are not fractals are self-similar. Notice the figure to the right. Notice that the outline of the figure is a trapezoid. Now look inside at all the trapezoids that make up the larger trapezoid. This is an example of ***self similarity.***

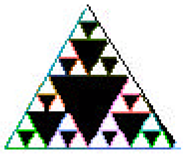


You can also think of self-similarity as copies. Each of the small trapezoids is a copy of the larger. Below are five other examples of self-similarity.

Self-Similarity of Fractals

To the right is the Sierpinski Triangle that we make in this unit. Notice that the outline of the figure is an equilateral triangle. Now look inside at all the equilateral triangles. Remember that there are infinitely many smaller and smaller triangles inside. How many different sized triangles can you find? All of these are similar to each other and to the original triangle - ***self similarity***



See all the copies of the original triangle inside? How many copies do you see where the ratio of the outer triangle's sides to the inner ones is 2:1? 4:1? 8:1? I think we have a pattern here. Can you find it?

Questions on Self-Similarity

Question 1: If the red image is the original figure, how many similar copies of it are contained in the blue figure?

Question 2: Are squares self-similar? (Can you form bigger squares out of smaller ones?) Are hexagons? (Can you form larger hexagons out of smaller ones?) Draw examples to justify your answer.

Question 3: Are circles similar? Are they self-similar?(Can you form larger circles out of smaller ones? Draw examples to justify your answer.

Question 4: Experiment with designing another self-similar figure.

Robert Devaney has more information on self-similarity.

You may obtain a print version of this page.

lanius@math.rice.edu

These pages were developed through GirlTECH, a teacher training and student technology council program sponsored by the Center for Research on Parallel Computation (CRPC), a National Science Foundation Science and Technology Center.